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# MUON STORAGE RINGS FOR 6D PHASE-SPACE COOLING

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*Abstract*

We describe several storage ring designs for reducing the 6-dimensional phase space of circulating muon beams. These rings utilize quadrupole and dipole magnets as well as wedge-shaped, liquid-hydrogen, energy-loss absorbers and energy compensating rf cavities. We obtain evaluations of their cooling performance by particle tracking simulation. Such rings are potentially useful for future Neutrino Factories or Muon Colliders as well as for existing facilities in which cooled, intense muon beams could enhance their physics programs.

## INTRODUCTION

Two key components of a future collider complex based on counter-circulating muon beams [1] are the target/capture system [2] and the cooling system[3]. In addition, cooling systems[4] are envisioned for Neutrino Factory complexes[5, 6] which are based on the collection and storage of circulating muon beams.

Attempts to reduce the cooling infrastructure, thereby reducing costs of the cooling system, have led to a consideration of cooling rings as a possible simplifying approach to achieve 6D cooling of the muon beam phase space. Whereas linear cooling systems are useful for cooling the transverse emittance of muon beams, cooling rings have the additional advantage of generating momentum dispersion in the circulating beam thus providing the opportunity to insert wedge absorbers into the beam in order to also reduce the longitudinal beam emittance. We discuss in this paper one such approach in which the ring structure is composed of quadrupole and dipole magnets along with the inclusion of energy-loss absorbers and rf cavities which are required to restore the energy of the muon beam [7].

## RING LATTICES

In general, our approach has been to obtain linear lattice solutions using the code SYNCH [8] and then transferring the lattice parameters to the code ICOOL [9] where the effects of energy loss in absorbers and the subsequent energy recovery in rf cavities can be modeled.

The system can cool both transversely and longitudinally, but the longitudinal cooling, which is proportional to the product of the absorber wedge angle and beam dispersion, causes a reduction of the cooling in the transverse bend plane[10]. The cooling, together with heating from scattering and straggling in the absorbers, results in a minimum achievable equilibrium emittance[11]. The expression for the transverse equilibrium emittance is given by

$$\epsilon_{N,eq} = \frac{\beta_{\perp} {E_s}^2}{2\beta m_\mu c^2 L_R (dE/ds)} \qquad (1)$$

where $\beta_{\perp}$ is the Courant-Snyder focusing parameter, $E_s$ is the characteristic scattering energy ($\sim$ 13.6 MeV), $\beta$ is the Lorentz velocity, $m_\mu$ is the muon mass, c is the velocity of light, $L_R$ is the material radiation length, and $dE/ds$ is the energy loss rate of the beam in the absorbing material. The equilibrium emittance sets the minimum achievable emittance and is directly proportional the beam parameter $\beta_{\perp}$.

Each cell is designed to obtain beam waists at the centers of both the absorbers and rf cavities. We seek a $\beta_{min}$ in the absorber which will yield a small equilibrium emittance. We also attempt to minimize the $\beta_{max}$ of the lattice, thus maximizing the transverse acceptances.

### *A Quadrupole-dipole lattice*

This 32 meter circumference ring has eight symmetric cells (Fig. 1) . The 22.5° dipoles and the adjacent center

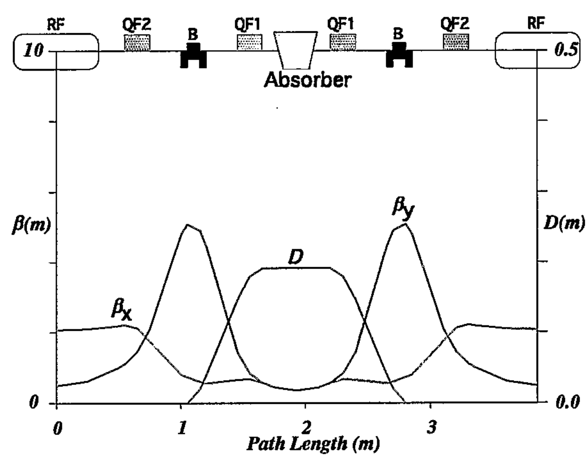


Figure 1: A quadrupole-dipole 8-cell ring lattice with $\beta_{min}$ and dispersion values in the wedge absorbers chosen to produce 6D cooling.

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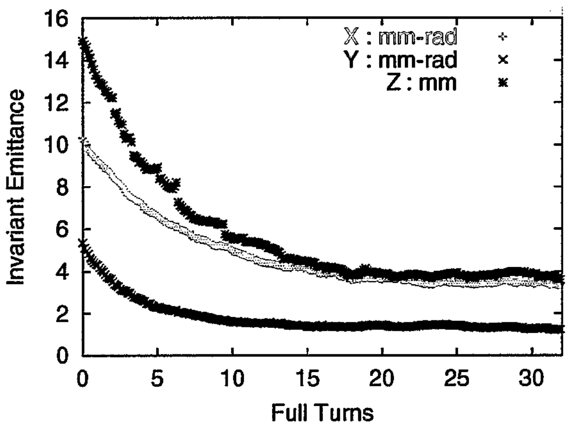


Figure 2: History of the normalized emittance for each plane as a function of full-ring turns in the dipole-quadrupole ring of Fig. 1

quadrupoles create finite dispersion in the absorber with zero dispersion in the rf cavities. The gradients in the quadrupoles and dipoles are adjusted to give $\beta_{min}$=25 cm at the cell center. We set the length of the absorber equal to the value of $\beta_{min}$ so that the $\beta_{\perp}$ parameter of the beam remains nearly constant within the absorber. For computational convenience in the simulations, the gradient dipole was replaced by a zero-gradient dipole bordered by two thin quadrupoles. Fig. 2 shows the performance of the ring in terms of the evolution of the normalized beam emittances. Note that an emittance reduction is achieved for all three planes.

## *Dipole-only lattice*

A particularly simple lattice design, shown as a 4-cell, 10-meter ring (Fig. 3), has only a single dipole magnet in each half cell. The desired orbit functions are obtained with the aid of edge focusing. In this case, the dispersion is non-zero everywhere, including in the rf cavities. The compactness of the ring and the low beta values give good beam acceptance and high cooling efficiency. Similar rings with three and two cells have also been designed and simulated, with comparable results in cooling performance. The performance of this ring (Fig. 4) is characterized by a merit factor, which is defined as the ratio of the initial 6D normalized emittance to the final emittance times the survival rate of the circulating muons (see Eq. 2). Muon losses are incurred both from dynamical losses in transit around the ring and losses from the decay of the muons.

$$Merit = Transmission \times \frac{\epsilon_{x_i}}{\epsilon_{x_f}} \frac{\epsilon_{y_i}}{\epsilon_{y_f}} \frac{\epsilon_{y_i}}{\epsilon_{y_f}} \qquad (2)$$

We observe, for this lattice design, a merit factor increase of $\sim$100 which corresponds to an increase in the muon beam phase-space occupation density by 2 orders of magnitude.

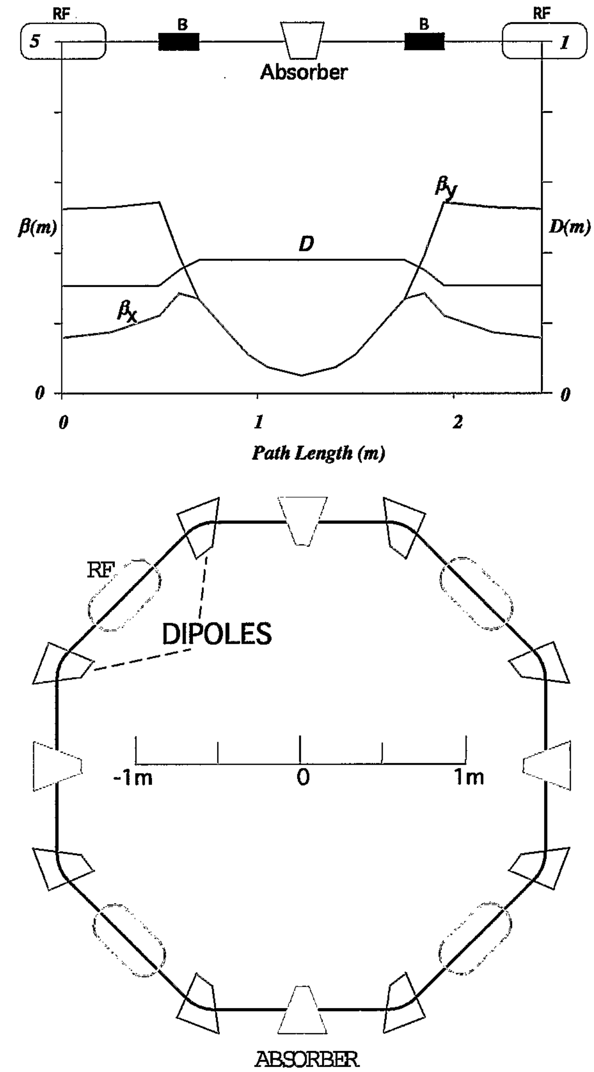


Figure 3: A dipole-only 4-cell ring lattice. The required focusing is accomplished by adjusting the dipole entrance and exit pole-face angles.

## *Reverse-bend dipoles*

The eight cells of this 29 meter ring each contain four dipoles, the center two of which have reverse bending (Fig. 5). The dipoles have zero gradient but strong edge focusing. The dispersion is zero at the ends of the cell. The main advantage of this feature, as in the previously described quadrupole-dipole lattice case, is that it allows for the inclusion of straight sections to the ring thus providing for the possibility of injection and extraction sections. Another potential advantage is that the circumference of the ring is increased thereby allowing for the injection and extraction of longer bunch trains.

The cooling performance (Fig. 6) of this ring, as measured by the merit factor (Eq. 2) is comparable to the previously considered dipole-only lattice with no reverse bends.

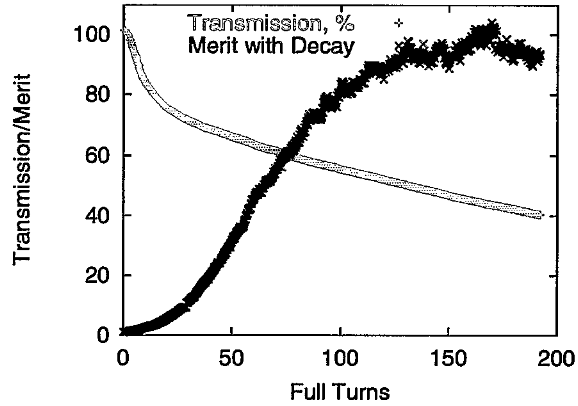


Figure 4: Performance (see Eq. 2 for Merit definition) of a 500 MeV/c circulating muon beam in the 4-cell, dipole-only ring of Fig. 3.

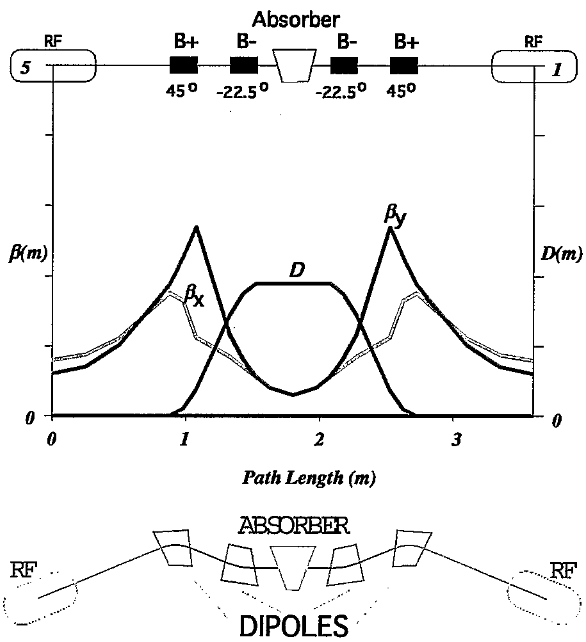


Figure 5: A dipole-only, 8-cell ring lattice including reverse bends. The pole-face angles and the reverse bending give 6D cooling as well as zero dispersion in the rf drift spaces.

## SUMMARY

We have demonstrated that the cooling of muon beams in 6D phase space can be achieved with storage ring containing magnetic elements consisting only of quadrupoles and dipoles and in some cases only dipoles. This can result in a significant simplification of the muon cooling system component of future muon colliders and/or neutrino factories. Our analysis is valid for linear optic designs. In the future we will extend these results to consider the non-linear effects of magnetic elements.

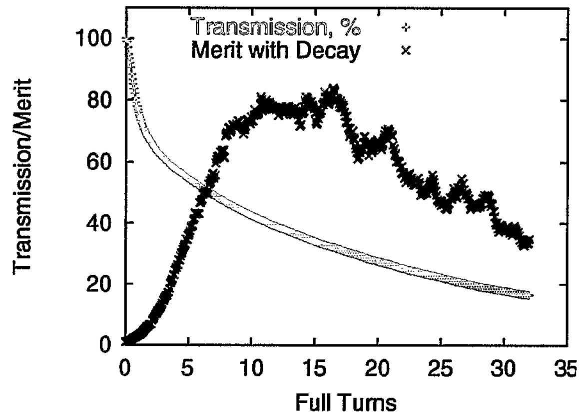


Figure 6: Performance of a 250 MeV/c beam circulating in the 8-cell, reverse-bend dipole ring of Fig. 5

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